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Radiation of Some Fractal Structures

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The fractal properties of a candle flame and fractal discharges are considered. The fractal concept is applied to ball lightning.

1. INTRODUCTION

The introduction of fractal ideas in physics by Mandelbrot (1977, 1982) has led to a new understanding of many physical phenomena. The development of these concepts has shown that along with discontinuous matter, porous physical systems with a fractal structure are widespread. Fractal ideas are becoming significant in contemporary physics (Stauffer and Stanley, 1990). Below we will consider three examples of the formation of fractal structures in physical processes: the radiation of a candle flame, the radiation of gas discharges resulting from the formation of fractal aggregates, and the properties of a fractal tangle such as a skeleton of ball lightning.

2. RADIATION OF A CANDLE FLAME

The analysis of the radiative parameters of a candle flame shows that its radiation is created by fractal aggregates. General information or fractal structures and fractal aggregates (Family and Landau, 1984; Stanley and Ostrowsky, 1985; Herrmann, 1986; Pietronero and Tosatti, 1986; Jullien and Botet, 1987; Meakin, 1986; Jullien *et al.*, 1988; Feder, 1988; Pietronero, 1988; Vicsek, 1987/1991; Smirnov, 1990a, 1991a; Smith, 1991; Family and Vicsek, 1991) and simple models for the radiation of fractal aggregates allow us to analyze the qualitative character of the processes under con-

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sideration. The absorption cross section of a small spherical macroscopic particle has the form (Landau and Lifshitz, 1960)

$$\sigma = \pi r_0^2 \frac{r_0}{\lambda} f_\omega \tag{1}$$

where r_0 is the particle radius, which is small compared with the wavelength of the photon $\lambda = 2\pi c/\omega$ (c is the light velocity, ω is the photon frequency), and the factor f_ω is expressed through the real and imaginary parts of the dielectric constant of the particle material. This parameter for soot particles and for the optical range of the spectrum is equal to $f_\omega = 1.00 \pm 0.06$ (Smirnov, 1988).

Within the framework of the simplest model of the radiation of fractal aggregates we take the boundary conditions for scattering of electromagnetic waves to be the same as in the case of separate spherical particles. Then we have for the absorption cross section of a fractal aggregate formula (1), where the particle radius r_0 is changed by the fractal aggregate radius R:

$$\sigma = f_\omega * \pi R^2 * R/\lambda \tag{2}$$

The number of individual particles in the fractal aggregate is equal to

$$n = (R/r_0)^D \tag{3}$$

where D is the fractal dimension of the fractal aggregate. Thus the radiation power of the fractal aggregate per unit mass of a sample is more $(R/r_0)^D$ times that for separate particles.

The spectral power p_ω of the radiation of an individual spherical particle with the temperature T is equal to

$$p_\omega = j_\omega \sigma \tag{4}$$

where $j_\omega = \hbar\omega^3[\exp(\hbar\omega/T) - 1]^{-1}/(4\pi^2 c^2)$ is the spectral radiation flux of a black body; σ is the absorption cross section of the particle. For the number density N of radiant particles the spectral power of radiation per unit volume is equal to Np_ω.

It has long been known that the glow of a candle flame is created by soot particles (see, for example, Faraday, 1957). This means that carbon of the flame is transformed partly into soot. Let us label the soot part of carbon by ξ and assume that small particles of soot are spherical. Further we calculate the value ξ on the basis of measured radiative parameters of the candle flame.

Let us analyze the measured radiative parameters of a candle flame, which are the spectral radiation flux I_ω and the spectral power P_ω of radiation. One also can determine the rate of fuel combustion in the candle flame, and from this the optimal content of carbon in the flame. These parameters of the candle flame were measured by Luisova *et al.* (1989, 1990) and we use the obtained values. Below we connect these values with the amount of soot in the candle flame, using the large value of the parameter $\hbar\omega/T$ ($\hbar\omega$ is the photon energy, T is the flame temperature), which is ~ 10 for optical photons. Therefore the luminosity of the flame is determined by a region near the hottest point of the flame. For simplicity we consider an isotropic model of the candle flame. Then the flame temperature near the hottest point of the flame has the form

$$T(r) = T_0 - \alpha r^2 \tag{5}$$

where T_0 is the maximal flame temperature, and r is the distance from the hottest point of the flame.

Let us calculate the radiative parameters of the transparent flame. Each soot particle is an isotropic radiator which creates the spectral power of radiation p_ω. The strongest dependence of p_ω on coordinates is determined by the factor $\exp(-\hbar\omega/T)$. On the basis of this dependence we have for the spectral power of radiation

$$\begin{aligned} P_\omega &= \int N p_\omega \, d\mathbf{r} \\ &= N p_\omega(T_0) \int \exp\left(-\frac{\hbar\omega}{T_0^2}\alpha r^2\right) d\mathbf{r} \\ &= N p_\omega(T_0) \left(\frac{\pi T_0^2}{\hbar\omega\alpha}\right)^{3/2} \end{aligned} \tag{6}$$

The spectral radiation flux is equal to

$$I_\omega = \int N p_\omega \frac{d\mathbf{r}}{4\pi r^2} = \frac{N p_\omega(T_0)}{4} \left(\frac{\pi T_0^2}{\hbar\omega\alpha}\right)^{1/2} \tag{7}$$

We assume in this formula that the focus of an optical system is directed to the hottest point during the measurement of the spectral radiation flux I_ω that corresponds to its maximal value. Then the ratio of the spectral power P_ω to the spectral radiation flux I_ω gives an effective area of the luminous surface that corresponds to this frequency ω:

$$S_\omega = P_\omega / I_\omega = 4\pi (T_0^2/\hbar\omega\alpha) \tag{8}$$

Thus, according to formulas (1), (4), and (6), the spectral power of radiation has the form

$$P_\omega = Np_\omega (S_\omega/4)^{3/2} = Nr_0^3(\pi f_\omega/\lambda)\, j_\omega (S_\omega/4)^{3/2} \tag{9}$$

The maximal flame temperature can be found on the basis of the dependence of I_ω on the frequency [according to (6), $I_\omega \sim \omega^{3/2} \exp(-\hbar\omega/T)$]. Another method of temperature determination involves with the addition of the salt NaCl to the flame. Then the spectral radiation flux of the centers of sodium resonance lines ($\lambda = 589.592$ and 588.995 nm) corresponds to the blackbody flux. These methods give the temperature of the candle flame as (Luisova *et al.*, 1989, 1990)

$$T_0 = 1800 \pm 50 \text{ K} \tag{10}$$

The effective surface area for the green region of the optical spectrum is $S_\omega = 2.6 \pm 0.5 \text{ cm}^2$, which gives according to formula (8) $\alpha = 600 \pm 150 \text{ K/cm}^2$, and

$$Nr_0^3 = 10^{-5.3 \pm 0.3} \tag{11}$$

The chemical scheme of the combustion process for stearin has the form

$$C_{18}H_{36}O_2 + 26O_2 \rightarrow 18CO_2 + 18H_2O \tag{12}$$

An energy of 39.7 kJ is released per 1 g of stearin at the optimal conditions of the process. Then the total combustion of stearin and total use of oxygen takes place, and 1 g of stearin corresponds to 14 g of air, or 1 g of carbon corresponds to 19 g of air. Assuming that the released energy is spent on heating the forming mixture only, we obtain the maximal flame temperature $T = 2500$ K. The measured flame temperature $T_0 = 1800 \pm 50$ K takes place at these conditions if approximately 70% of the released energy is spent on air heating and the rest, 30%, can be transformed to radiation.

Let us check the assumption that the flame is transparent. Introduce the optical density of the flame τ as the ratio of the spectral brightness of radiation I_ω to the spectral brightness of a black body at the temperature T_0. We have for the hottest region of the candle flame:

$$\tau = I_\omega / j_\omega = (\lambda_0/\lambda)^{1/2} \tag{13}$$

where on the basis of measurements $\lambda_0 = 20 \text{ nm} \times 10^{\pm 0.6}$. In particular, for the green region of the optical spectrum, the optical density of the candle flame is $\tau = 0.2 \times 10^{\pm 0.3}$.

The highest flame radiation takes place at optimal conditions which correspond for the temperature (10) to the following carbon density in the flame:

$$\rho_C = (1.4 \pm 0.1) \times 10^{-5}\ \mathrm{g/cm^3} \tag{14}$$

Assuming the carbon density in soot to be the same as for graphite (2.1 $\mathrm{g/cm^3}$), we have from this for the parameter Nr_0^3

$$Nr_0^3 = (1.6 \pm 0.1) \times 10^{-6}\xi$$

Comparing this with (11), which follows from the measured parameters, we have

$$\xi = 4 \times 10^{\pm 0.4} \tag{15}$$

Thus we obtain that the amount of soot in the candle flame exceeds the optimal concentration of fuel in the hot region. This means that the observed radiation power of a candle flame exceeds the optimal one at the temperature (10). The only way to overcome this contradiction is to assume that soot particles are joined into fractal aggregates. This increases the radiation output of soot per unit mass by $(R/r_0)^{3-D}$ times. Note that carbon fractal aggregates are known to form in flames (Nelson, 1989).

Let us make estimations using the parameters of carbon fractal aggregates (Ershov *et al.*, 1990) formed as a result of an explosion of material with a content of carbon. The parameters of these fractal clusters of soot are $r_0 = 3$ nm, $R = 20$ nm, and $\mathrm{D} = 1.9$. For these parameters we have $(R/r_0)^{3-D} = 9$. Then the parameter $\xi = 40\% \times 10^{\pm 0.4}$ instead of formula (16), and the disagreement disappears.

Thus one can conclude that the formation of carbon fractal aggregates in a candle flame explains its observed radiative parameters, while assuming that the soot particles are solid spheres leads to lower values of the specific power of a candle than is observed. This shows the importance of fractal concepts for flame processes and the need for their detailed analysis.

3. FRACTAL DISCHARGES

Fractal discharges according to their definition contain fractal aggregates which create their luminosity. Usually fractal discharges are considered as laboratory analogs of ball lightning. For this reason we consider some observed properties of ball lightning (Smirnov, 1987a, 1990b, 1992, 1993) which are of interest for the problems under consideration.

Mean ball lightning has a rigid skeleton and a spotty glowing structure with a typical temperature of the hot regions of ~ 2000 K, while the mean temperature of air inside the skeleton is 350–500 K. The luminosity of mean ball lightning is the same as for an electrical lamp of power 100–150 W, and the mean lifetime of mean ball lightning is $8 \times 10^{\pm 0.3}$ sec. Usually the lifetime of fractal discharges is 0.1–1 sec. One can see that all the parameters of fractal discharges except the last one correspond to the parameters of ball lightning. But according to their lifetimes fractal discharges are not analogs of ball lightning, and we will consider fractal discharges independently of ball lightning.

Fractal aggregates in fractal discharges are formed from metallic vapor which appears in the discharge region as a result of vaporization of electrodes under the influence of discharge currents. Therefore various types of excitation can create fractal discharge, such as high-frequency discharge (Powell and Finkelstein, 1970; Manjikin and Shachparonov, 1991), electric breakdown in air (Barry, 1968; Andrianov and Sinitzyn, 1977; Ofuruton, 1989), power arc discharge (Silberg, 1965, 1978; Golka, 1988; Dijkhuis, 1985, 1988; Avramenko *et al.*, 1990; Igolkin and Savelyev, 1992), radiofrequency discharge (Corum and Corum, 1989, 1990), underwater arc discharge (Golubnichij *et al.*, 1991; Golka, 1991, 192), corona discharge, etc. Unfortunately, fractal aggregates have not been detected in any of these experiments because of a lack of understanding of its nature. In addition, detailed analysis is lacking in most cases, and it is impossible to prove that the radiation of fractal discharges is created by fractal aggregates. An exception is the experiment by Powell and Finkelstein (1970). Though this experiment was performed before the creation of the concept of fractal aggregates, it contains many results which allow us to reproduce the physical picture of processes in this discharge. We analyze these processes on the basis of the Powell–Finkelstein experiment.

Let us describe briefly the Powell and Finkelstein (1970) experiment and its results. A 75-MHz arc was used in gases at a pressure of 0.5–3 atm. After excitation the discharge was cut off, luminosity occurred in a 15-cm Pyrex tube which was connected with a discharge region. Note the following important properties of this afterglow:

1. The luminosity arose a small time (<0.1 sec) after excitation was cut off.

2. The curves of radiant power decay were approximately exponential with a time constant of 0.2–0.3 sec.

3. The gas temperature was 2000–2500 K, as measured by the electrical resistance of a fine tungsten wire in the center of the luminosity.

4. At the beginning the specific power of radiation was of the order of 10 W/l for visible (0.4–0.72 μm) and infrared (0.72–1.1 μm) radiation. The power of the visible radiation was rather larger than the power of infrared radiation.

5. The radiation parameters depend strongly on the type of gas. Air, O_2, N_2, and N_2O were used as a discharge gas. The effect was observed in all the gases, but in N_2 it was weak, and in N_2O it was strong. The color of the luminosity depended on the sort of gas.

6. There was a strong dependence of the effect on the electrode material. The luminosity was observed for Pt, Au, Ag, Cu, Zn, Cd, Sn, W, and Al electrodes. All are materials with a high melting point and not easily vaporized. For easily vaporized electrodes, such as Hg and Pb, the luminosity was weak and was characterized by a small time constant, $\sim$0.05 sec.

The measurements of the Powell–Finkelstein experiment allow us to analyze the physical picture of the process. We use the following scheme. As a result of the action of the electric current the electrodes are vaporized to a small degree, and metallic vapor in the form of atoms is an admixture to the discharge gas. After discharge is cut off, atoms are condensed to small particles, and later these particles join into fractal aggregates (Family and Landau, 1984; Stanley and Ostrowsky, 1985; Herrmann, 1986; Pietronero and Tosatti, 1986; Jullien and Botet, 1987; Meakin, 1986; Jullien *et al.*, 1988; Feder, 1988; Pietronero, 1988; Vicsek, 1989/1991, Smirnov, 1990a, 1991a; Smith, 1991; Family and Vicsek, 1991). This creates the luminosity of the hot gas, which later decreases with cooling of the discharge gas.

We make some estimates within the framework of the above scheme using the results of the Powell–Finkelstein experiment. We take air as the discharge gas, Pt as the evaporating material, and use the following values of the parameters: $T = 2000$ K, $P = 10$ W/l, $r_0 = 3$ nm, $R = 1$ μM, and the fractal dimension of fractal aggregates $D = 1.7$. Then the estimates give the following results:

1. The concentration of Pt in air which provides the observed power of radiation is equal to 10^{-4} g Pt to g air if Pt is found in the form of separate particles in the discharge region, and this value is equal to 10^{-8} g/g if Pt is in the form of fractal aggregates.

2. Assuming that the air cooling is determined by radiation only, we obtain the typical time τ of change of the power $(dP/dt = -P/\tau)$

$$\tau = c_p T/5P$$

where c_p is the specific heat capacity of air. For the above parameters, $\tau \sim 10$ sec, i.e., cooling is related to convection processes. One can expect for other discharge setups higher times of luminosity, up to ~ 10 sec.

3. A typical time of formation of fractal aggregates under the above conditions due to the cluster–cluster aggregation mechanism (Meakin, 1983; Kolb *et al.*, 1983) is ~ 0.01–0.1 sec according to the simplest model of the aggregation process (Smirnov, 1990a, 1991a).

Note that from the analysis of the Powell–Finkelstein experiment it follows that along with the above processes, chemical processes are also possible. This can enhance the luminosity and change its color, as was observed for the cases of O_2 and N_2O.

Thus there is a correspondence between the Powell–Finkelstein experiment and the origin of fractal aggregates in the afterglow which creates the luminosity of discharge after its excitation is cut off. One can expect that the understanding of the nature of fractal discharges will allow one to study the processes inside these discharges in detail. Such a study will give a new understanding of the physics of such discharges.

4. FRACTAL CONCEPT OF BALL LIGHTNING

Fractal concepts are important in understanding the nature of ball lightning. Studies of ball lightning using fractal concepts have shown that both in nature and in laboratory devices fractal structures with specific physical properties exist. Investigations of such objects are important for understanding new situations in physics.

Let us consider the fractal concept of ball lightning, which has developed as new analogs of the properties and processes of ball lightning have been used. At the first stage of the studies the rigid skeleton of ball lightning was assumed to be a system of threadlike aerosols (Alexandrov *et al.*, 1982a), based on experiments on the explosion of metallic wires (Alexandrov *et al.*, 1982b). Later, the concept of fractal aggregates was used (Smirnov, 1986), which led to the consideration of an aerogel structure (Smirnov, 1987b) and a structure of interwoven fractal fibers (Smirnov, 1991b). The last version of the fractal concept of ball lightning used the experience in producing fractal fibers as a result of the laser irradiation of surfaces (Lushnikov *et al.*, 1990). The current version of ball lightning is as follows (Smirnov, 1991b). The rigid skeleton has the structure of a fractal tangle consisting of interwoven fractal fibers. A separate fractal fiber consists of bounded nanometer particles, and the mean specific gravity of fractal fibers is two—three orders of magnitude smaller than that of condensed matter. Fractal fibers occupy ~ 0.01 part of the fractal tangle volume. Therefore the specific gravity of a fractal tangle is four—five orders of magnitude smaller than that corresponding to the condensed substance.

The fractal concept of ball lightning explains many of its properties. Ball lightning has gaseous, liquid, and solid properties because ball lightning has a rigid skeleton with a small surface tension and low specific gravity. Such a combination of properties explains its different mechanical properties. Indeed, the possibility to float is connected with its small specific gravity: the ability to pass through small holes and narrow slits is explained by its small surface tension; and its elastic properties as it recoils from hard surfaces result from its rigid skeleton.

An important property of a fractal tangle is the phase transition tangle–globule, which is like that in a polymer fiber with self-intersections (Flory, 1971; de Gennes, 1977). For mean ball lightning this is expected at 700 ± 200 K (Smirnov, 1991b). At low temperatures a fractal tangle has a spherical form, at high temperatures it can have any form and can change. Note that ball lightning has a spherical form in 90% of observations and in 1% of the observations transitions band–ball or ball–band take place (Grigorjev *et al.*, 1989). The transitions between different forms of ball lightning cannot be explained on the basis of a simple model.

The properties of importance are the energy processes in ball lightning. Because the ball lightning skeleton consists of nanometer particles, it has a large specific surface energy, ~ 1 kJ/g. The transformation of this energy into thermal energy as a result of increasing of the specific area of the internal surface can lead to heating of the substance up to temperatures of ~ 2000 K. The process of transformation of the internal energy of ball lightning takes place in the form of thermal waves (Smirnov, 1991c) which propagate along fractal fibers, and many separate thermal waves exist simultaneously.

Thus the structure of ball lightning and the energy process inside it explain the spotty structure of ball lightning glow (Smirnov, 1987a, 1990b, 1992, 1993) as a result of the heating of fractal fibers near the fronts of thermal waves and shows why the mean temperature inside ball lightning ($\leqslant 100$ K) diverges from the temperature of the hot zones (2000 K) which create the glowing. But the theory does not give a number of thermal waves. Introducing this value in the theory allows one to estimate the brightness and lifetime of mean ball lightning. The color of ball lightning is determined by the glowing admixtures which are found in hot zones.

The specific surface energy of a fractal tangle explains the internal energy of ball lightning as estimated from an analysis of observations (Smirnov, 1987a, 1990b, 1992, 1993). It is in accordance with the fact of ball lightning explosion which takes place as a result of the above processes of energy release, and the observation of heat sensation from ball lightning may be explained also by the parameters of the above energy processes.

As a system with hot zones, a glowing fractal tangle is a source of plasma. Thus ball lightning creates a conducting channel on its way that can cause the electric breakdown of air, like usual lightning. It can lead to large destruction which a lightning rod cannot protect against, and the released energy in this case exceeds remarkably the mean internal energy of the ball lightning.

The fractal concept explains various details of ball lightning. For example, the correlation between its diameter and lifetime (Smirnov, 1987a, 1993; Stakhanov, 1985) is explained by an increase of the time of thermal wave propagation with an increase of the size of the ball lightning. The fractal concept describes the evolution of ball lightning starting from its origin and related with electric processes near the surface. As a result of these processes a weakly ionized vapor is formed and is transformed subsequent to the skeleton of ball lightning after processes in this vapor involving condensation. Note that the time of formation of a fractal tangle is some minutes.

To understand some advantages and disadvantages of the current fractal versions of ball lightning, let us compare the preceding version of an aerogel-like structure with the current one of a fractal tangle structure. In the case of the fractal tangle structure of ball lightning it is possible to have many simultaneously propagating thermal waves, while for the aerogel-like structure only one spherical thermal wave can exist. Therefore the observational picture of many glowing zones is explained by the fractal tangle structure, and are not explained by the aerogel structure of the ball lightning skeleton. At the same time the fractal tangle version does not allow one to estimate a number of thermal waves, i.e., it demands subsequent modification, for example, by using the polymer structure (Bychkov, 1992).

Thus the fractal version of ball lightning is under development. As a result of studies of ball lightning we obtain a new physical object—a fractal tangle—which is of interest both for fundamental science and for applications. It has gaseous, liquid, and solid properties simultaneously, and specific radiative and explosive properties related to the fractal tangle. Therefore investigations of this object are important independent of the ball lightning problem.

5. CONCLUSION

In conclusion note on the basis of the above examples that the use of general fractal concepts (Mandelbrot, 1977, 1982) for physical processes and phenomena has changed our understanding of these phenomena.

REFERENCES

Alexandrov, V. Ya., Golubjev, E. M., and Podmoshenskij, I. V. (1982a). *Soviet Physics Technical Physics*, **2**, 1221.

Alexandrov, V. Ya., Borodin, I. P., Kichenko, E. V., and Podmoshenskij, I. V. (1982b). *Soviet Physics Technical Physics*, **2**, 527.

Andrianov, A. M., and Sinitzyn, V. I. (1977). *Soviet Physics Technical Physics*, **22**, 1342.

Avramenko, R. F., *et al.* (1990). *Zhurnal Tekhnicheskoi Fiziki*, **60**(12), 73.

Barry, J. D. (1968). *Journal of Atmospheric and Terrestrial Physics*, **3**, 313.

Bychkov, V. L. (1992). In *Proceedings 9th International Conference on Atmospheric Electricity*, St. Petersburg, Russia, p. 842.

Corum, K. L., and Corum, J. F. (1989). *Tesla Coil Builders Association News*, **8**, 13.

Corum, K. L., and Corum, J. F. (1990). *Uspekhi Fizicheskikh Nauk*, **160**(4), 47.

De Gennes, P. G. (1977). *Scaling Concepts in Polymer Physics*, Cornell University Press, Ithaca, New York.

Dijkhuis, G. C. (1985). *Nederlands Tijdschrift voor Natuurkunde*, **51B**, 125.

Dijkhuis, G. C. (1988). In *Proceedings 9th International Wroclaw Symposium on Electromagnetic Compatibility*, Wroclaw, Poland, p. 166.

Ershov, A. P., Kuperstokh, A. L., and Kolomijchuk, V. N. (1990). *Pisma Zhurnal Tekhnicheskoi Fiziki*, **16**(3), 43.

Family, F., and Landau, D. P. (1984). *Kinetics of Aggregation and Gelation*, North-Holland, Amsterdam.

Family, F., and Vicsek, T. (1991). *Dynamics of Fractal Surfaces*, World Scientific, Singapore.

Faraday, M. (1957). *The Chemical History of a Candle*, Crowell, New York.

Feder, J. (1988). *Fractals*, Plenum Press, New York.

Flory, P. (1971). *Principles of Polymer Chemistry*, Cornell University Press, Ithaca, New York.

Golka, R. K. (1988). In *Proceedings 9th International Wroclaw Symposium on Electromagnetic Compatibility*, Wroclaw, Poland, p. 59.

Golka, R. K. (1991). *NASA Conference Proceedings*, **II**, 3106.

Golka, R. K. (1992). In *Proceedings 9th International Conference on Atmospheric Electricity*, St. Petersburg, Russia, p. 854.

Golubnichij, P. I., Gromenko, V. M., and Krutov, Ju. M. (1991). In *Ball Lightning*, B. M. Smirnov, ed., IVTAN, Moscow [in Russian].

Grigorjev, A. I., Grigorjeva, I. D., and Shirjaeva, S. O. (1989). In *Science of Ball Lightning*, Y. H. Ohtsuki, ed., World Scientific, Singapore, p. 176.

Herrmann, H. J. (1986). *Physics Reports*, **136**, 155.

Igolkin, S. I., and Savelyev, S. K. (1992). In *Proceedings 9th International Conference on Atmospheric Electricity*, St. Petersburg, Russia, p. 830.

Jullien, R., and Botet, R. (1987). *Aggregation and Fractal Aggregates*, World Scientific, Singapore.

Jullien, R., Peliti, L., Rammal, R., and Bocara, N., eds. (1988). *Universalities in Condensed Matter*, Springer-Verlag, Berlin.

Kolb, M., Botet, R., and Jullien, R. (1983). *Physical Review Letters*, **51**, 1123.

Luisova, L. A., Smirnov, B. M., and Khahaev, A. D. (1989). *Soviet Physics Doklady*,, **34**, 1086.

Luisova, L. A., Smirnov, B. M., Khahaev, A. D., and Chugin, V. P. (1990). *High Temperature* (Russia), **28**, 674.

Landau, L. D., and Lifshitz, E. M. (1960). *Electrodynamics of Continuous Media*, Addison-Wesley, Reading, Massachusetts.

Lushnikov, A. A., Negin, A. E., and Pakhomov, A. V. (1990). *Chemical Physics Letters*, **161**, 138.

Mandelbrot, B. B. (1977). *Fractals: Form, Chance and Dimension*, Freeman, San Francisco.
Mandelbrot, B. B. (1982). *The Fractal Geometry of Nature*, Freeman, San Francisco.
Manjikin, E. A., and Shachparonov, I. M. (1991). In *Ball Lightning*, B. M. Smirnov, ed., IVTAN, Moscow, p. 68 [in Russian].
Meakin, M. (1983). *Physical Review Letters*, **51**, 1119.
Meakin, P. (1986). *Critical Reviews in Solid State and Materials Sciences*, **322**, 789.
Ohtsuki, Y. H., and Ofuruton, H. (1989). In *Science of Ball Lightning*, Y. H. Ohtsuki, ed., World Scientific, Singapore, p. 31.
Nelson, J. (1989). *Nature*, **339**, 611.
Pietronero, L., ed. (1988). *Fractals. Physical Origin and Properties*, Plenum Press, New York.
Pietronero, L., and Tosatti, E. (1986). *Fractals in Physics*, North-Holland, Amsterdam.
Powell, J. R., and Finkelstein, D. (1970). *Scientist*, **58**, 262.
Silberg, P. A. (1965). In *Problems of Atmospheric and Space Electricity*, S. C. Corotini, ed., Elsevier, Amsterdam, p. 435.
Silberg, P. A. (1978). *Journal of Applied Physics*, **49**, 1110.
Smirnov, B. M. (1986). *Soviet Physics Uspekhi*, **29**, 133.
Smirnov, B. M. (1987a). *Physics Reports*, **152**, 177.
Smirnov, B. M. (1987b). *Soviet Physics Uspekhi*, **30**, 133.
Smirnov, B. M. (1988). *The Problem of Ball Lightning*, Nauka, Moscow [in Russian].
Smirnov, B. M. (1990a). *Physics Reports*, **188**, 3.
Smirnov, B. M. (1990b). *Soviet Physics Uspekhi*, **33**, 261.
Smirnov, B. M. (1991a). *Physics of Fractal Clusters*, Nauka, Moscow [in Russian].
Smirnov, B. M. (1991b). *Soviet Physics Uspekhi*, **34**, 711.
Smirnov, B. M. (1991c). *Soviet Physics Uspekhi*, **34**, 526.
Smirnov, B. M. (1992). *Soviet Physics Uspekhi*, **35**.
Smirnov, B. M. (1993). *Physics Reports*, **35**.
Smith, J. M. (1991). *Fundamentals of Fractals for Engineers and Scientists*, Wiley, New York.
Stakhanov, I. P. (1985). *On the Physical Nature of Ball Lightning*, Energoatomizdat, Moscow [in Russian].
Stanley, H. E., and Ostrowsky, N., eds. (1985). *On Growth and Form*, Martinus Nijhoff, The Hague.
Staufer, D., and Stanley, H. E. (1990). *From Newton to Mandelbrot*, Springer, Berlin.
Vicsek, T. (1989/1991). *Fractal Growth Phenomena*, World Scientific, Singapore.